Progress Report

Study of the Relationship Between Frequency and Gas Temperature in Whistles

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Introduction

During the current report period, analytical work on determining a relationship between frequency and gas temperature in whistles has proceeded along two lines. A lumped parameter analysis of the whistle geometry shown in Figure 1 has been accomplished and is discussed below. Dimensional analysis studies have also been carried out and are continuing.

The primary goal of the study is to relate the frequency to the gas temperature, geometry, and blowing velocity. Initially, temperature has not been considered and emphasis both analytically and experimentally in the companion laboratory project on whistles has been placed on understanding the relationship between geometry, velocity and the frequencies produced. The particular geometric arrangement employed thus far is shown in Figure 1. Here, the interaction between the jet edge and the organ pipe column produces a complex acoustic field that has not yielded to exact analysis. Approximate techniques for prediction of frequency as a function of velocity are, however, discussed below.

Background Comments

As noted previously, the model shown in Figure 1 has two essential features: (1) the jet impinging on the edge of a plate and (2) the organ pipe. It is well known that the pitch will increase with blowing velocity for a geometry such as that in Figure 1. Lunn (1) states that this may be due to the kinematic stiffness of the blown jet and the consequent discontinuity of the pressure at the mouth due to the centrifugal force of curvilinear flow. The trends can be predicted by the equation given by Lunn as

$$\frac{\Delta f}{f} = \frac{\lambda^2}{4\pi^2 b^2 \ell} \qquad (1)$$

where $\Delta f/f$ is the relative increase in frequency; λ is the wavelength of mode of vibration in the pipe; ℓ is the length of the pipe; and b is a coefficient that depends on the blowing velocity and the geometric parameters and tends to account for the change in the effective length of the pipe column as it interacts with the jet. This concept is expanded and employed in later discussions.

Mercer (2) notes that the application of edge tone theories to the organ pipe must be made with caution. He states that the frequency generated by the edge tone is governed by the resonating column of the pipe and that the mouth-aperture changes the effective length of the pipe. He does not give an equation that relates corrected length to the variation in frequency. Jones (3) also discusses end correction factors as do Gaylord and Caster (4) who also present experimental data. They define the threshold pressure or velocity as the pressure or velocity at which coupled oscillation begins and is maintained. Accordingly, the threshold velocity is constant if h/ℓ remains constant, where h is the distance from the edge to the nozzle and ℓ is length of the resonating pipe.

According to Figure 2, the threshold velocity is the velocity at which the frequency of the edge tone becomes equal to the resonating frequency of pipe. In actual practice, coupled oscillation begins and is maintained earlier than the theoretical threshold velocity. According to Figure 3, the graph of the frequency generated by the organ pipe is asymptotic to the resonating frequency of the tube. As shown in Figure 3, there will be a jump in the frequency generated by the organ pipe when the frequency of the pure edge tone becomes equal to the first overtone of the pipe. The foregoing discussion is used as a basis for the following development.

Correction Factor

For the closed end pipe, the frequencies for the modes of vibration are given by

$$f = \frac{nc}{4\ell} \qquad (2)$$

where f = frequency, c = sound velocity, $n = 1, 3, 5, \ldots$, ℓ = length of pipe.

In Equation (2) there is always some end correction for the effective length of pipe.

Considering the organ pipe, sound of some frequency is generated due to jet edge system, and it works as a sound source for the rest of the pipe, and the pipe should resonate with the frequencies given by Equation (2). The edge tone phenomena is sensitive to the presence of any obstacle in the near field. Alan Powell (5) suggests, "A given flow pattern interacting with the edge, gives rise to certain motions contiguously to the edge which would not rise if the edge were absent." In the organ pipe, these motions will be carried inside the resonating pipe. The air inside the pipe will begin to vibrate and will disturb the motions produced at the edge. If the frequency produced in the jet edge system of the organ pipe is smaller than the resonating frequency of the pipe, the motions initiated due to the presence of the edge, will be suppressed by the resonating column of the pipe. But when the frequency of the edge tone becomes equal to or greater than the resonating frequency of the pipe, the disturbances produced by the edge will be strengthened. The same thing will happen when the frequency of the edge tone becomes equal to the overtone and so the jump will take place.

When the frequency of the edge tone becomes equal to the resonating frequency of the pipe, the motions initiated due to the presence of edge will be disturbed by the resonating column but sound of a given frequency will be generated. This phenomena, results in an increase of the effective length of pipe. Hence, some end correction must be introduced.

The end correction introduced here, is different from the end correction described before (after the equation (1)). This end correction is not fixed, but changes with change of frequency generated by the jet edge system. The end correction is inversely proportional to the frequency, or directly proportional to the wavelength. And whenever, the jump occurs in the frequency of organ pipe, a similar jump occurs in the end correction.

As the end correction is inversely proportional to frequency,

$$\Delta \ell \ \alpha \ \frac{n}{fe}$$

where $\Delta \ell$ = change in effective length

$n = 1, 3, 5, \ldots$

fe = frequency of edge tone.

So that $$\Delta \ell = \frac{nK}{fe}$$

and the effective length is $\ell' = \ell + \Delta\ell = \ell + \frac{nk}{fe}$.

Thus the frequencies for the modes of vibration, for the organ pipe, may be given by

$$f = \frac{nc}{4(\ell + \frac{nk}{fe})} \qquad (3)$$

where

f = resonating frequency

c = velocity of sound

K = constant (end correction factors)

ℓ = length of organ pipe

fe = frequency of the edge tone

This formula is for closed pipe only. If the other end is not closed then the formula will be in the following form.

$$f = \frac{nc}{4(\ell + \frac{nk}{fe} + \ell e)}$$

where

ℓe = end correction depending upon the end not being excited

n = 1, 3, 5, ...

or 2, 4, 6, ... depending upon the other end condition.

The end correction factor K depends upon the geometry of the organ pipe, i.e., the width of the nozzle (or mouth), the distance of the edge from the nozzle, area of the nozzle, area of organ pipe. To predict the frequency generated by the organ pipe, K should be found from the experimental data. As an example consider a closed pipe 4 inches long at room temperature. The fundamental pipe mode will occur $f_p = 672$ Hz. The end correction can then be found from experimental data if the measured frequency f_g is known. For the arrangement of Figure 1, the measured value was 624 Hz. The correction factor is then K = 258 Hz-in.

The theoretical value for the first jump in frequency can then be predicted as

$$f_g = \frac{}{4(\ell + \frac{K}{fe})} = 655 \text{ Hz}$$

where

$$fe = 3f_\rho = 2016 \text{ Hz}$$

Experimental data gives a value of 655 Hz for the first jump. Similar computations for the second jump gives 1870 Hz as compared to a measured value of 1803 Hz.

Summary

In organ pipe, the presence of jet edge system in front of the pipe, affects the effective length of pipe and so some end correction must be introduced which changes with change in the edge tone frequency. This end correction cannot completely be related with velocity of air as the exact relation between velocity of air and edge tone frequency is not yet known.

References

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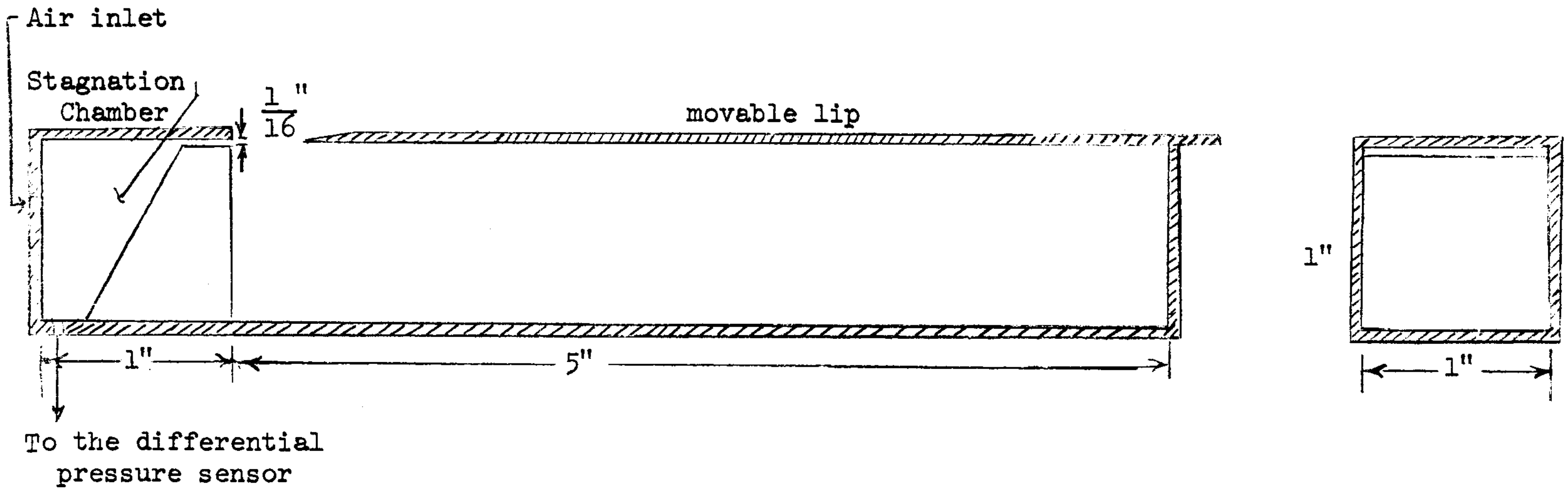

Figure 1. Organ pipe

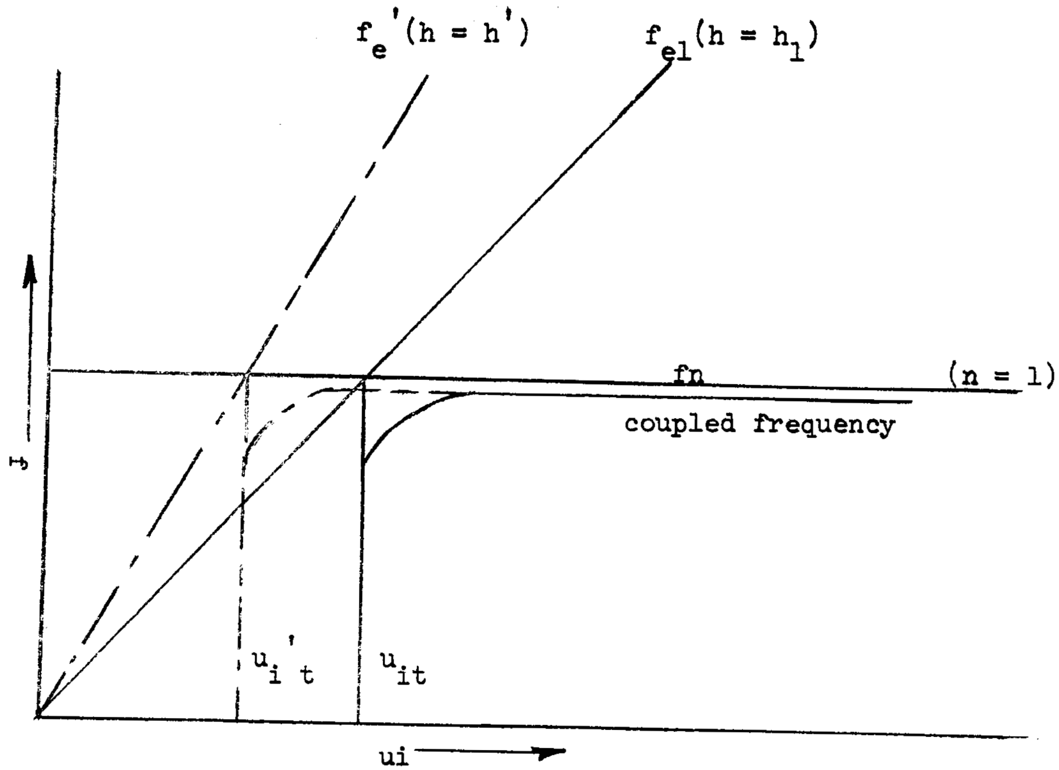

Figure 2. Frequency as a function of input velocity.

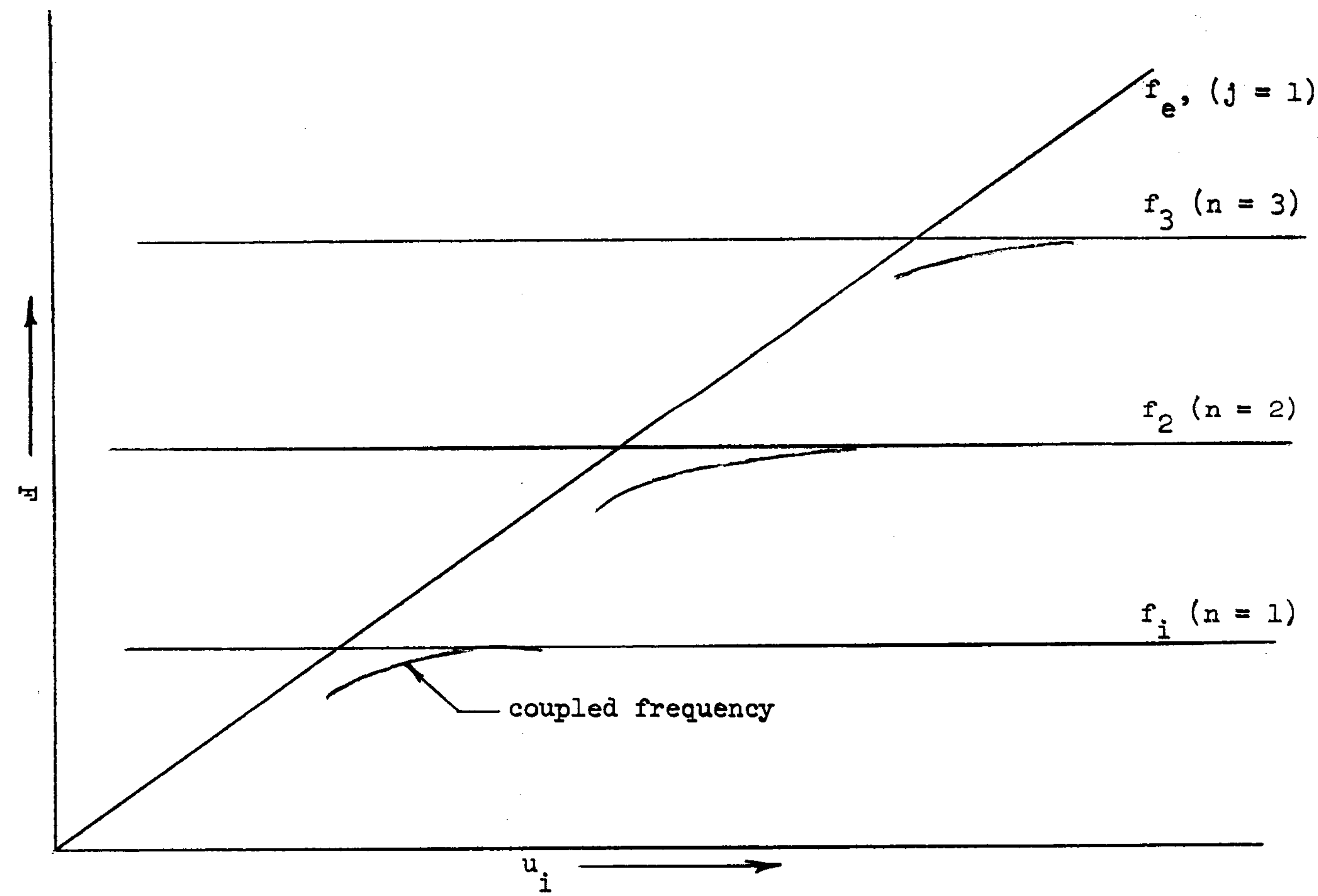

Figure 3. Frequency versus velocity for a jet edge and resonator system.